RADFORD REVIEW

The Aon Hewitt Multiple-Point Black-Scholes Model

The vast majority of companies develop estimates for the fair value of employee stock options using the traditional Black-Scholes model; however, this widely accepted pricing model is not without its flaws. One of the key weaknesses of the Black-Scholes model lies in how companies traditionally collapse their robust distribution of option activity into one, single-point estimate of expected life.

This single-point estimate provides the Black-Scholes model with a single timeframe by which all participants are expected to exercise (or cancel, post-vesting) their options. Yet, in the real world, options are generally exercised or cancelled within a range of timeframes around the single-point estimate. Therefore, using a single-point expected life results in a less accurate fair value calculation. In fact, single-point calculations create an error rate ranging from 1% to 8% higher than a true distribution of real employee behavior. Thus, given the mandate by ASC Topic 718 to use historical data to set assumptions for future exercise behavior, shouldn't we seek ways to use historical data in a more robust and accurate fashion?[1]

One solution to this problem is to consider every option exercise as a data point for input into the Black-Scholes model, thereby preserving the unique distribution of option activity throughout the history of a company. This approach is called the *Aon Hewitt Multiple-Point Black-Scholes Model*. In essence, this method creates multiple exercise points, which, when taken together, form a distribution determining an assumed expected life. To make this approach a reality, companies will need to perform a separate valuation for each historical grant based on the actual life of that option. Furthermore, historical data for outstanding (i.e., unexercised) options is also considered. Individual valuations will be performed for outstanding options, each with some assumed date of exercise (discussed later). The individual valuation of each historical option will be based on current Treasury rates and current volatility estimates specific to the actual or assumed term of the historical option.

Before completing an analysis of historical activity, a company must estimate when their currently outstanding options will be exercised. The most common method is to assume exercise at the midpoint between the current date (if vested) or the vesting date (if unvested) and the expiration date. This approach is consistent with the simplified approach as outlined by the Securities and Exchange Commission (SEC) in Staff Accounting Bulletins #107 and #110.

The fair value developed for purposes of the current grant is developed as a weighted average of the individual valuations described above. Given the appropriate use of historical option grants in setting assumptions

[1] ASC Topic 718.10.55.55-33 states that "*estimating the fair value of an option based on a single expected term that effectively averages the differing exercise and post-vesting employment termination behaviors of identifiable groups of employees will potentially misstate the value of the entire award.* "

regarding current fair values, the *Aon Hewitt Multiple-Point Black-Scholes Model* is an improvement over the traditional single fixed-point Black-Scholes model, providing more accuracy and defensibility.

Impact of Using Actual and Assumed Exercise Behavior on Fair Value

As described above, a multiple-point model accommodates a distribution of actual and assumed individual exercise behavior. Because it values each historical option separately, its aggregate effect is to employ term structures for the risk-free rate, the dividend yield, and expected volatility assumptions, further increasing the accuracy of the fair value results.

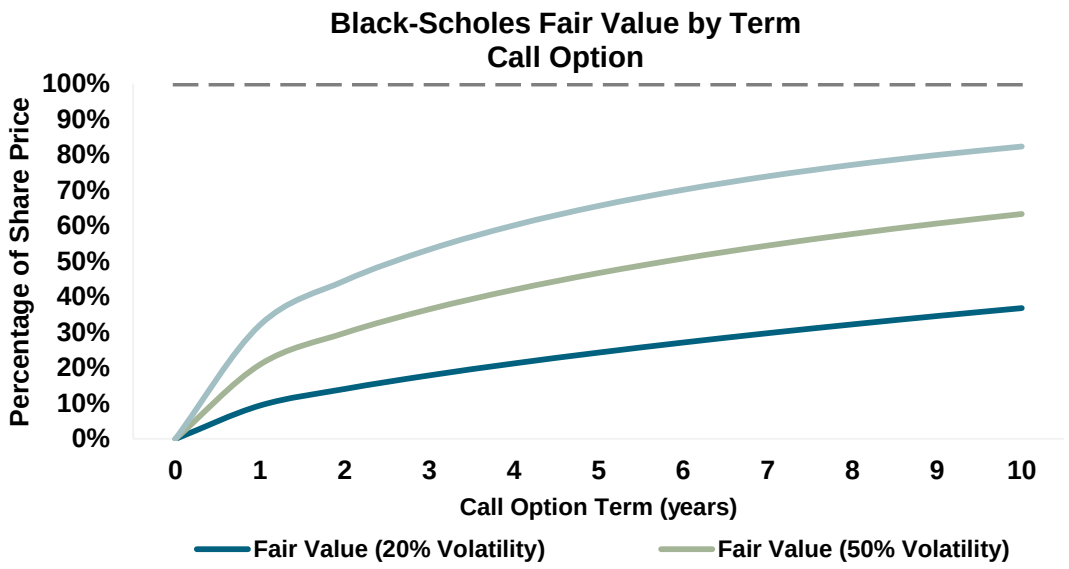
To illustrate this, the example below shows actual exercise behavior and assumptions for outstanding grants. The valuation date is assumed to be Jan. 1, 2011, and outstanding options are assumed to be exercised consistent with the process described previously. Each data point within the distribution yields a fair value based on the time until activity, and current Treasury rates, and expected dividend yield.

	Grant Date	Job Level	Number of Options	Vesting Date	Activity Date	Assumed Activity Date	Life of Option	Risk-Free Rate	Expected Volatility	Expected Dividend Yield	Fair Value
Exercised/Cancelled Options	1/1/2001	Executive	1,000	1/1/2005	5/1/2010	N/A	9.33	4.26%	30.00%	0.00%	48.43%
	4/1/2002	Non-Executive	100	4/1/2006	6/1/2008	N/A	6.17	3.71%	30.00%	0.00%	37.47%
	7/1/2005	Non-Executive	300	7/1/2009	1/1/2011	N/A	5.50	3.54%	30.00%	0.00%	34.84%
	2/1/2006	Non-Executive	500	2/1/2010	7/1/2008	N/A	2.41	2.27%	30.00%	0.00%	20.72%
	2/1/2006	Non-Executive	500	2/1/2010	1/1/2010	N/A	3.92	2.96%	30.00%	0.00%	27.94%
	9/1/2006	Executive	1,000	9/1/2010	10/1/2007	N/A	1.08	1.46%	30.00%	0.00%	13.10%
	9/1/2006	Non-Executive	100	9/1/2010	1/1/2011	N/A	4.33	3.15%	30.00%	0.00%	29.86%
	2/1/2007	Executive	10,000	2/1/2011	5/1/2010	N/A	3.24	2.66%	30.00%	0.00%	24.80%
	2/1/2007	Executive	1,000	2/1/2011	5/1/2008	N/A	1.25	1.57%	30.00%	0.00%	14.16%
	1/1/2001	Executive	10,000	1/1/2005	2/1/2002	N/A	1.08	1.48%	30.00%	0.00%	13.12%
	1/1/2001	Executive	10,000	1/1/2005	7/1/2010	N/A	9.49	4.28%	30.00%	0.00%	48.94%
	1/1/2001	Non-Executive	300	1/1/2005	1/1/2002	N/A	1.00	1.43%	30.00%	0.00%	12.56%
	1/1/2001	Non-Executive	500	1/1/2005	1/1/2003	N/A	2.00	2.09%	30.00%	0.00%	18.57%
	1/1/2001	Non-Executive	100	1/1/2005	1/1/2004	N/A	3.00	2.59%	30.00%	0.00%	23.68%
	1/1/2001	Non-Executive	100	1/1/2005	1/1/2005	N/A	4.00	3.01%	30.00%	0.00%	28.35%
	1/1/2001	Non-Executive	300	1/1/2005	1/1/2006	N/A	5.00	3.39%	30.00%	0.00%	32.75%
	1/1/2001	Non-Executive	500	1/1/2005	1/1/2007	N/A	6.00	3.68%	30.00%	0.00%	36.83%
	1/1/2001	Executive	1,000	1/1/2005	1/1/2008	N/A	7.00	3.89%	30.00%	0.00%	40.58%
	1/1/2001	Non-Executive	300	1/1/2005	1/1/2009	N/A	8.00	4.07%	30.00%	0.00%	44.11%
	3/1/2008	Non-Executive	200	3/1/2012	1/1/2011	N/A	2.84	2.49%	30.00%	0.00%	22.86%
Outstanding Options	1/1/2001	Executive	1,000	1/1/2005	N/A	12/31/2010	10.00	4.35%	30.00%	0.00%	50.49%
	4/1/2002	Non-Executive	100	4/1/2006	N/A	8/16/2011	9.37	4.27%	30.00%	0.00%	48.58%
	7/1/2005	Non-Executive	300	7/1/2009	N/A	4/1/2013	7.75	4.03%	30.00%	0.00%	43.26%
	2/1/2006	Non-Executive	500	2/1/2010	N/A	7/17/2013	7.46	3.97%	30.00%	0.00%	42.20%
	2/1/2006	Non-Executive	500	2/1/2010	N/A	7/17/2013	7.46	3.97%	30.00%	0.00%	42.20%
	9/1/2006	Executive	1,000	9/1/2010	N/A	10/31/2013	7.16	3.92%	30.00%	0.00%	41.18%
	9/1/2006	Non-Executive	100	9/1/2010	N/A	10/31/2013	7.16	3.92%	30.00%	0.00%	41.18%
	2/1/2007	Executive	10,000	2/1/2011	N/A	1/16/2014	6.96	3.87%	30.00%	0.00%	40.40%
	2/1/2007	Executive	1,000	2/1/2011	N/A	1/16/2014	6.96	3.87%	30.00%	0.00%	40.40%
	3/1/2008	Non-Executive	200	3/1/2012	N/A	7/31/2014	6.41	3.77%	30.00%	0.00%	38.42%
	Fixed Point Estimates		**52,500**				**5.31**	**3.48%**	**30.00%**	**0.00%**	**34.03%**
	Weighted Average		**52,500**				5.31				32.36%
	Valuation Delta by Incorporating a Distribution of Behavior										**(4.91%)**

For simplicity, the example above does not illustrate the use of a term structure on the expected volatility or the dividend yield for each individual option; however, it does illustrate the use of a yield curve on the risk-free rate of return.

Maintaining the distribution of exercise behavior in this example creates a 4.91% reduction in fair value, creating a stronger valuation that is more reflective of actual events. This reduction occurs for two reasons:

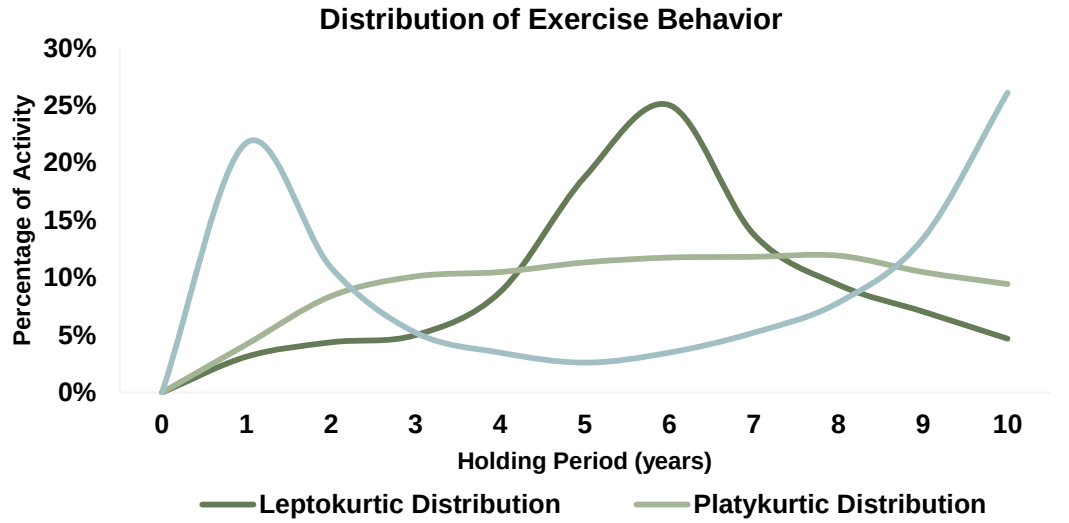
(1) Inherent in option pricing theory is the principal of an increasing decay rate, such that the rate at which the fair value increases decreases as the holding period increases (described in ASC 718 10.55.33-34). See the graph below for an illustration of this principal based on volatilities of 20%, 50%, and 80%.



(2) During periods of increasing yield curves, activity occurring earlier creates lower risk-free rates, and therefore lower valuations. In low inflation periods, it is rare to see decreasing yield curves.

The amount by which the *Aon Multiple-Point Black-Scholes Model* will reduce fair value is therefore dependent on the distribution of exercise behavior, which is summarized by the kurtosis of the distribution. Generally, we see three types of distributions:

i. A high-peaked (*leptokurtic*) distribution, in which most exercise occurs around the same time;
ii. A uniform (*platykurtic)* distribution, in which exercise occurs evenly throughout the contractual term; and
iii. A bimodal distribution, such that exercise occurs normally right after vesting, and then prior to the contractual term (at the beginning and at the end).



The leptokurtic and platykurtic curves are most commonly observed among companies with sufficient historical data. The bimodal curve is rarer, although it is occasionally seen.

To estimate the *Aon Hewitt Multiple Point Black-Scholes* fair value, we combine the three distribution curves (leptokurtic, platykurtic, and bimodal) with the three volatility curves (low, medium, and high).

			Leptokurtic Distribution	Platykurtic Distribution	Bimodal Distribution
		Single-Point Black-Sholes	Discount	Discount	Discount
Volatility	20% (Low)	26.81%	-1.8%	-2.8%	-6.1%
	50% (Medium)	50.34%	-2.4%	-3.8%	-8.2%
	80% (High)	69.65%	-2.8%	-4.3%	-9.5%

The results clearly illustrate the importance of utilizing the detailed records for the historical analysis of option activity rather than collapsing the distribution to one, single-point estimate..

To further accentuate the potential discount, we studied the results of applying the *Aon Hewitt Multiple-Point Black-Scholes* model for 20 actual companies, which the following table illustrates. These companies vary by number of employees, stock price, number of options granted, and industry. The second and third columns show fair values as a percentage of grant prices. The reduction in fair value (fourth column) is derived by taking the difference between the results in columns two and three, and dividing by the result in column two.

	Model Comparison			Valuation Assumptions					Distribution Analysis	
Company	Fixed Point Black-Scholes	Aon Multiple-Point Black-Scholes	Fair Value Reduction	Expected Life	Contractual Term	Volatility	Dividend Yield	Risk-Free Rate	Standard Deviation	Kurtosis
Company A	13.28%	12.26%	(7.65%)	4.29	10.00	16.11%	3.36%	4.44%	2.3590	(0.3778)
Company B	36.33%	35.14%	(3.28%)	5.84	10.00	35.04%	1.00%	3.86%	2.3883	(0.9050)
Company C	44.89%	43.29%	(3.57%)	4.89	10.00	46.46%	0.00%	3.76%	2.2066	(1.0080)
Company D	19.08%	18.00%	(5.68%)	4.79	10.00	24.95%	3.67%	4.32%	2.4140	(1.0919)
Company E	55.82%	54.28%	(2.76%)	5.05	10.00	61.06%	0.00%	4.12%	1.8318	0.0940
Company F	31.02%	30.30%	(2.33%)	6.19	10.00	22.98%	0.59%	4.36%	1.8428	(0.5975)
Company G	42.39%	40.53%	(4.38%)	4.07	10.00	47.19%	0.00%	4.46%	2.1376	(0.6557)
Company H	59.50%	56.28%	(5.40%)	5.26	10.00	64.71%	0.00%	4.47%	2.4881	2.0704
Company I	55.99%	53.77%	(3.95%)	4.74	10.00	63.45%	0.00%	4.32%	2.0822	2.4096
Company J	37.19%	36.80%	(1.04%)	3.61	5.00	43.00%	0.00%	4.36%	1.0037	7.2713
Company K	49.44%	48.02%	(2.87%)	3.91	7.00	60.78%	0.00%	3.97%	1.5042	3.0813
Company L	62.34%	59.27%	(4.92%)	5.11	10.00	70.77%	0.00%	4.45%	2.1348	2.7479
Company M	56.73%	54.31%	(4.26%)	5.81	10.00	56.44%	0.00%	4.47%	1.9833	2.3470
Company N	48.78%	44.90%	(7.96%)	3.81	10.00	59.95%	0.00%	4.37%	2.4418	1.5674
Company O	29.83%	29.38%	(1.50%)	4.93	6.00	29.38%	1.20%	4.39%	0.8175	5.7420
Company P	54.23%	52.06%	(4.00%)	5.30	10.00	57.30%	0.00%	3.87%	2.2993	2.2516
Company Q	43.17%	42.31%	(1.99%)	5.69	10.00	38.55%	0.00%	4.14%	1.9894	2.6006
Company R	67.55%	64.43%	(4.61%)	5.17	10.00	79.74%	0.00%	4.34%	2.1908	2.6212
Company S	17.02%	16.37%	(3.87%)	4.23	10.00	21.34%	3.20%	4.38%	1.8860	2.5333
Company T	28.65%	27.88%	(2.67%)	5.31	10.00	28.56%	1.65%	4.34%	1.9172	2.3716
Average	42.66%	40.98%	(3.93%)	4.90	9.40	46.39%	0.73%	4.26%	1.9959	1.7537
Average	42.66%	40.98%	(3.94%)	4.90	9.40	46.39%	0.73%	4.26%	1.9959	1.7537

**Note: Options granted with terms shorter than ten years produce lower reductions in fair value over the single fixed-point Black-Scholes model, as they typically have more leptokurtic curves.*

From our point of view, we connote reductions in fair value as the "error" produced by using a single fixed-point Black-Scholes model vs. a multiple-point model. Aon Hewitt observes that the fixed-point Black-Scholes model creates the greatest error for companies with some or all of the following characteristics:

- High dividend yields;
- Short historical average life;
- Large standard deviations of exercise behavior; and/or
- Small or negative kurtoses.

Of the given criteria, a large standard deviation and small kurtosis are the strongest indicators that a valuation model should consider the distribution of exercise behavior in order to accurately determine fair value.

Advantages and Disadvantages of the *Aon Hewitt Multiple-Point Black-Scholes Model*

Upgrading from the traditional Black-Scholes model to the *Aon Hewitt Multiple-Point Black-Scholes Model* will almost certainly improve the accuracy and defensibility of employee stock option valuations. However, the model is more complex, forcing companies weigh the pros and cons of the multiple-point approach.

On a positive note, the multiple-point model provides flexibility in valuation by allowing companies to carve out data for certain subsets of populations. For example, if options are only granted to select groups of employees, this approach can easily accommodate the elimination of certain data to better reflect the valuation of the options for the population currently receiving grants. Similarly, it is easier to reflect any expectations of change in the future due to other demographic characteristics such as age, gender, or locality.

Furthermore, one of the criticisms of the traditional Black-Scholes model is that it is not tailored to accurately value American-style options with vesting periods, such as employee stock options. The *Aon Hewitt Multiple-Point Black-Scholes Model* addresses this criticism, as it values each historical option based on the terminal point (either actual or assumed) of its life. In other words, it values each historical option as a plain European-style option, which is what the traditional Black-Scholes model was originally created to do.

Of course, the weakness of any Black-Scholes model is that exercise behavior continues to only be a function of time. As we know, the decision to exercise is also affected by other variables such as stock price increases and behavioral factors. Indeed, to develop valuations that are even more accurate, it is necessary to use more sophisticated valuation techniques to develop exercise behavior as a function of time and stock price movements.

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