The Aon Multiple-Point Black-Scholes Model

Description of the Model

The vast majority of companies that have disclosed the fair value of employee stock options under FAS 123 have used the traditional Black-Scholes model. One of the weaknesses of this model is the use of a single-point estimate of expected life. This expected life assumes that all options are exercised or cancelled at that single, fixed point in time. This single, fixed point is usually based on an average of all historical exercises and cancellations. As we know, in the real world, options are generally exercised or cancelled with some distribution around that single, fixed-point estimate. Using a single fixed-point expected life results in a less accurate fair value calculation. Given the mandate by FAS 123(R) to use historical data to set assumptions for future exercise behavior, the goal is to make this data as useful as possible in assessing exercise behavior.

One solution to this problem is to value each individual historical exercise and cancellation using the Black-Scholes model and the actual time to exercise. In essence, this approach creates multiple exercise points, which taken together form a distribution for setting the assumed expected life. A separate valuation will be performed for each historical grant based on the actual life of that option. Further, historical data for outstanding (i.e. unexercised) options is also considered. Individual valuations will be performed of outstanding options, each with some assumed date of exercise. The individual valuation of each historical option will be based on current Treasury rates and current volatility estimates specific to the actual or assumed term of the historical option.

Before performing these valuations, it is necessary to set an assumed date of exercise for outstanding options. It is common to assume that outstanding options are exercised at one of the following:

(1) 25% of the remaining term after vesting. For example, an option that has been held for 4 years of its 10-year term would be assumed to be exercised at 5.50 years;

(2) 50% of the remaining term after vesting. For example, an option that has been held for 4 years of its 10-year term would be assumed to be exercised at 7.00 years. This approach would be consistent with the simplified approach as outlined by the Securities & Exchange Commission in Staff Accounting Bulletin #107. This approach is conservative, since historical options that have been held for 4 years take into account vesting restrictions, and in this example vesting restrictions would no longer apply in the future;

(3) 50% of the remaining term after the valuation date. For example, an option subject to three-year cliff vesting that has been held for two years of its 10-year term would be exercised at 6.00 years.

The fair value developed for purposes of the current grant is developed as a weighted average of the individual valuations described above. Given the appropriate use of historical option grants in setting assumptions regarding current fair values, the *Aon Multiple-Point Black-Scholes Model* is an improvement over the traditional single fixed-point Black-Scholes model, with respect to accuracy and defensibility.

Impact of Using Actual and Assumed Exercise Behavior on Fair Value

As described above, this model allows for modeling of the distribution of actual and assumed individual exercise behavior. Because it values each historical option separately, its aggregate effect is to employ term structures for the risk-free rate, the dividend yield, and expected volatility assumptions. This further increases the accuracy of the fair value results.

To illustrate this, the example below shows grants that have been exercised and are still outstanding. The valuation date for the current grant is assumed to be 1/1/2006. Outstanding options are assumed to be exercised at the midpoint of the remaining term as described in (3) above.

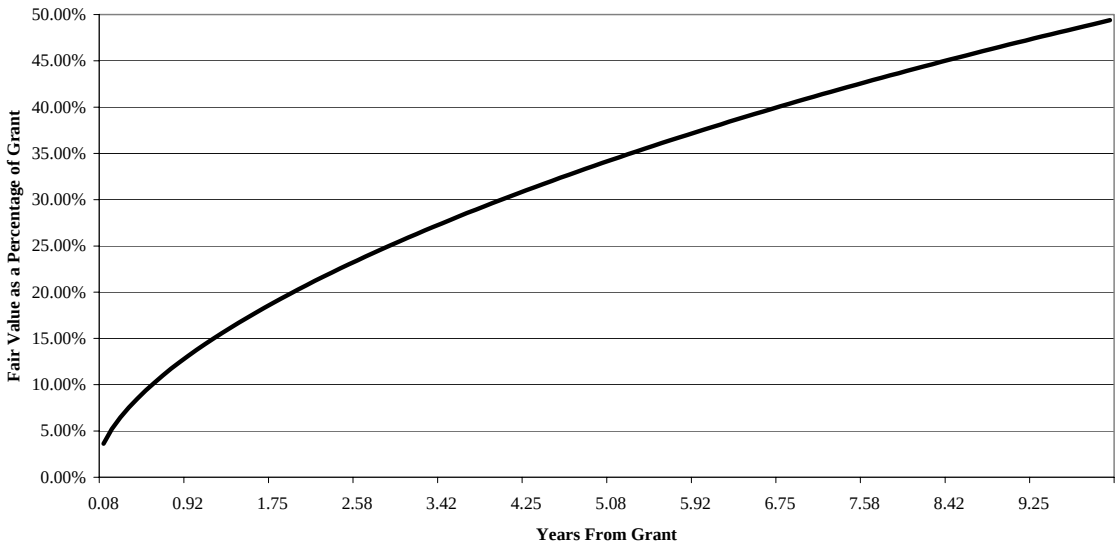
	Grant Date	Job Level	Number of Options	Exercise Date	Assumed Exercise Date	Life Of Option	Risk-Free Rate	Expected Volatility	Expected Div. Yield	Fair Value
Exercised Options	1/1/1996	Executive	1,000	5/1/2005	N/A	9.33	4.26%	30.00%	0.00%	48.43%
	4/1/1997	Non-Executive	100	6/1/2003	N/A	6.17	3.71%	30.00%	0.00%	37.45%
	7/1/2000	Non-Executive	300	1/1/2006	N/A	5.50	3.54%	30.00%	0.00%	34.84%
	2/1/2001	Non-Executive	500	7/1/2003	N/A	2.41	2.27%	30.00%	0.00%	20.71%
	2/1/2001	Non-Executive	500	1/1/2005	N/A	3.92	2.96%	30.00%	0.00%	27.94%
	9/1/2001	Executive	1,000	10/1/2002	N/A	1.08	1.46%	30.00%	0.00%	13.10%
	9/1/2001	Non-Executive	100	1/1/2006	N/A	4.33	3.15%	30.00%	0.00%	29.85%
	2/1/2002	Executive	10,000	5/1/2005	N/A	3.24	2.66%	30.00%	0.00%	24.81%
	2/1/2002	Executive	1,000	5/1/2003	N/A	1.24	1.57%	30.00%	0.00%	14.14%
	1/1/1996	Executive	10,000	2/1/1997	N/A	1.09	1.48%	30.00%	0.00%	13.14%
	1/1/1996	Executive	10,000	7/1/2005	N/A	9.50	4.28%	30.00%	0.00%	48.95%
	1/1/1996	Non-Executive	300	1/1/1997	N/A	1.00	1.43%	30.00%	0.00%	12.58%
	1/1/1996	Non-Executive	500	1/1/1998	N/A	2.00	2.07%	30.00%	0.00%	18.57%
	1/1/1996	Non-Executive	100	1/1/1999	N/A	3.00	2.57%	30.00%	0.00%	23.66%
	1/1/1996	Non-Executive	100	1/1/2000	N/A	4.00	3.01%	30.00%	0.00%	28.35%
	1/1/1996	Non-Executive	300	1/1/2001	N/A	5.00	3.39%	30.00%	0.00%	32.76%
	1/1/1996	Non-Executive	500	1/1/2002	N/A	6.00	3.68%	30.00%	0.00%	36.83%
	1/1/1996	Executive	1,000	1/1/2003	N/A	7.00	3.89%	30.00%	0.00%	40.59%
	1/1/1996	Non-Executive	300	1/1/2004	N/A	8.00	4.07%	30.00%	0.00%	44.10%
	3/1/2003	Non-Executive	200	1/1/2006	N/A	2.84	2.49%	30.00%	0.00%	22.87%
Outstanding Options	1/1/1996	Executive	1,000	N/A	12/31/2005	10.00	4.35%	30.00%	0.00%	50.50%
	4/1/1997	Non-Executive	100	N/A	8/16/2006	9.38	4.27%	30.00%	0.00%	48.59%
	7/1/2000	Non-Executive	300	N/A	4/1/2008	7.75	4.03%	30.00%	0.00%	43.25%
	2/1/2001	Non-Executive	500	N/A	7/17/2008	7.46	3.97%	30.00%	0.00%	42.20%
	2/1/2001	Non-Executive	500	N/A	7/17/2008	7.46	3.97%	30.00%	0.00%	42.20%
	9/1/2001	Executive	1,000	N/A	10/31/2008	7.17	3.92%	30.00%	0.00%	41.19%
	9/1/2001	Non-Executive	100	N/A	10/31/2008	7.17	3.92%	30.00%	0.00%	41.19%
	2/1/2002	Executive	10,000	N/A	1/16/2009	6.96	3.87%	30.00%	0.00%	40.41%
	2/1/2002	Executive	1,000	N/A	1/16/2009	6.96	3.87%	30.00%	0.00%	40.41%
	3/1/2003	Non-Executive	200	N/A	7/31/2009	6.42	3.77%	30.00%	0.00%	38.44%
Fixed Point Estimate			52,500			5.31	3.48%	30.00%	0.00%	34.04%
Average			52,500			5.31				32.36%
Incremental Difference in Valuation by Incorporating a Distribution of Behavior										**-4.91%**

Each exercised and outstanding grant is valued based on the individual life of the option and current treasury rates commensurate with the actual life (for exercised and cancelled options) and assumed expected life (for unexercised options).

For simplicity, the example above does not illustrate the use of a term structure on the expected volatility or the dividend yield for each individual option. However, it does illustrate the use of a yield curve on the risk-free rate of return when looking at all historical grants in aggregate.

Creating a distribution of exercise behavior in this example, results in a 4.91% reduction in fair value compared to traditional Black-Scholes – a 4.91% reduction that is clearly seen as the result of a *better* valuation. This reduction will occur for two reasons:

(1) Inherent in option pricing theory is the principal of an increasing decay rate. This means that the rate at which the fair value increases *decreases* as the holding period increases (described in Paragraph A30 of FAS 123(R)). See the graph below for an illustration of this principal based on a volatility of 30% and a risk-free rate of 4%.



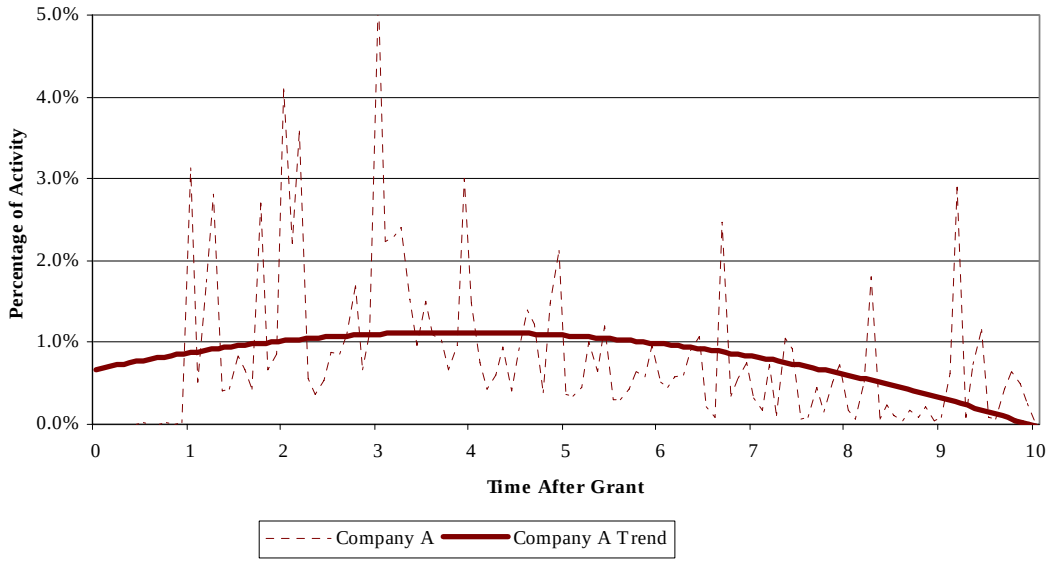
(2) During periods of positive yield curves, the multiple-point distribution creates lower average risk-free rates, and therefore lower valuations. (In low inflation environments, it is rare to see protracted periods of negative yield curves.)

The amount by which the *Aon Multiple-Point Black-Scholes Model* will reduce fair value will therefore be dependent on the distribution of exercise behavior, as seen in the kurtosis of the distribution. A flat-topped distribution curve (*platykurtic* curve) would

create greater reductions in fair value. A high-peaked curve (*leptokurtic* curve) would create smaller reductions in fair value.

An extreme example of a leptokurtic curve could occur if a company grants options with a relatively short contractual term (i.e., five or six years) and a strict vesting period (i.e., four-year cliff). This results in a very short time frame in which options can be exercised, and this distribution of behavior would yield a high-peaked curve. This type of exercise behavior distribution results in little fair value differentiation between the fixed-point Black-Scholes model and the *Aon Multiple-Point Black-Scholes Model*.

The following chart illustrates the distribution of exercise behavior over time for one sample company. The expected life of 4.29 years has a standard deviation of 2.359 and a kurtosis of –0.3778. The standard deviation and kurtosis suggests that a fixed-point Black-Scholes model is inappropriate because the exercise activity is significantly spread out over the term. The trend curve below is an example of a platykurtic curve. The *Aon Multiple-Point Black-Scholes Model* results in a fair value that is 7.65% lower than the fixed-point Black-Scholes model.



The following table illustrates the analysis of exercised, cancelled, and outstanding options for twenty companies. These companies vary by number of employees, stock price, number of options granted, and industry. The second and third columns show the fair values as a percentage of the grant prices. The reduction in fair value (fourth column) is derived by taking the difference between the results in columns two and three, and dividing by the column two result.

Company	Fixed Point Black-Scholes	*Aon Multiple-Point Black-Scholes*	Reduction in Fair Value	Expected Life	Contractual Term	Volatility	Dividend Yield	Risk-Free Rate	Expected Life Analysis Standard Deviation	Kurtosis
Company A	13.28%	12.26%	-7.65%	4.29	10.00	16.11%	3.36%	4.44%	2.3590	(0.3778)
Company B	36.33%	35.14%	-3.28%	5.84	10.00	35.04%	1.00%	3.86%	2.3883	(0.9050)
Company C	44.89%	43.29%	-3.57%	4.89	10.00	46.46%	0.00%	3.76%	2.2066	(1.0080)
Company D	19.08%	18.00%	-5.68%	4.79	10.00	24.95%	3.67%	4.32%	2.4140	(1.0919)
Company E	55.82%	54.28%	-2.76%	5.05	10.00	61.06%	0.00%	4.12%	1.8318	0.0940
Company F	31.02%	30.30%	-2.33%	6.19	10.00	22.98%	0.59%	4.36%	1.8428	(0.5975)
Company G	42.39%	40.53%	-4.38%	4.07	10.00	47.19%	0.00%	4.46%	2.1376	(0.6557)
Company H	59.50%	56.28%	-5.40%	5.26	10.00	64.71%	0.00%	4.47%	2.4881	2.0704
Company I	55.99%	53.77%	-3.95%	4.74	10.00	63.45%	0.00%	4.32%	2.0822	2.4096
Company J	37.19%	36.80%	-1.04%	3.61	5.00	43.00%	0.00%	4.36%	1.0037	7.2713
Company K	49.44%	48.02%	-2.87%	3.91	7.00	60.78%	0.00%	3.97%	1.5042	3.0813
Company L	62.34%	59.27%	-4.92%	5.11	10.00	70.77%	0.00%	4.45%	2.1348	2.7479
Company M	56.73%	54.31%	-4.26%	5.81	10.00	56.44%	0.00%	4.47%	1.9833	2.3470
Company N	48.78%	44.90%	-7.96%	3.81	10.00	59.95%	0.00%	4.37%	2.4418	1.5674
Company O	29.83%	29.38%	-1.50%	4.93	6.00	29.38%	1.20%	4.39%	0.8175	5.7420
Company P	54.23%	52.06%	-4.00%	5.30	10.00	57.30%	0.00%	3.87%	2.2993	2.2516
Company Q	43.17%	42.31%	-1.99%	5.69	10.00	38.55%	0.00%	4.14%	1.9894	2.6006
Company R	67.55%	64.43%	-4.61%	5.17	10.00	79.74%	0.00%	4.34%	2.1908	2.6212
Company S	17.02%	16.37%	-3.87%	4.23	10.00	21.34%	3.20%	4.38%	1.8860	2.5333
Company T	28.65%	27.88%	-2.67%	5.31	10.00	28.56%	1.65%	4.34%	1.9172	2.3716
Average	42.66%	40.98%	-3.94%	4.90	9.40	46.39%	0.73%	4.26%	1.9959	1.7537

We connote the reduction in fair value as the error produced by using the fixed-point Black-Scholes model. Aon has observed that the fixed-point Black-Scholes model creates the greatest error for companies that have some or all of the following characteristics:

- High dividend yields
- Short historical average life
- Large standard deviations of exercise behavior
- Small or negative kurtoses

Note that the options granted with terms shorter than ten years produce lower reductions in fair value over the fixed point Black-Scholes model.

The last two criteria, a large standard deviation and small kurtosis, are the strongest indicators that a valuation model must consider the distribution of exercise behavior in order to accurately determine the fair value.

Advantages and Disadvantages of the Aon Multiple-Point Black-Scholes Model

The *Aon Multiple-Point Black-Scholes Mode*l will always result in a lower fair value than the traditional Black-Scholes model, given that the corresponding fixed-point expected life was properly selected as the mean of the distribution of observed exercises.

The *Aon Multiple-Point Black-Scholes Model* allows for additional flexibility in valuation. First, it is easier to carve out data for certain subsets of populations. For example, if options are only granted to select groups of employees, this approach can easily accommodate the elimination of certain data to better reflect the valuation of the options for the population currently receiving grants. Similarly, it is easier to reflect any expectations of change in the future due to other demographic characteristics such as age, gender, or locality.

Finally, one of the criticisms of the traditional Black-Scholes model has been that it is not tailored to accurately value American style options with vesting periods such as employee stock options. The *Aon Multiple-Point Black-Scholes Model* addresses this criticism since it values each historical option based on the terminal point (either actual or assumed) of its life. In other words, it values each historical option as a plain European option, which is what traditional Black-Scholes was created to do.

Of course, the weakness of any Black-Scholes model is that exercise behavior continues to only be a function of time. As we know, the decision to exercise is also affected by other variables such as stock price increases and behavioral factors. Indeed, to develop valuations that are even more accurate, it is necessary to use more sophisticated valuation techniques to develop exercise behavior as a function of time and stock price movements.